Parallel Lines Notes

Let's review:

Slope	y-intercept	Slope-Intercept Form
$m = \frac{y_2 - y_1}{x_2 - x_1}$	$(0, b)$ $x = 0$	$f(x) = mx + b$ or $y = mx + b$

Example 1: Write the equation of a line that passes through the points $(-4, -6)$ and $(0, 10)$.

$m = \frac{10 - -6}{0 - -4} = \frac{16}{4} = 4$ y-intercept : $(0, 10)$

$\boxed{y = 4x + 10}$

Example 2: Write the equation of a line that that has a slope of -2 and passes through the point $(-7, 9)$.

$m = -2$

$y = -2x + b$

$9 = -2(-7) + b$

$9 = 14 + b$

$-14 \quad -14$

$b = -5$

$\boxed{y = -2x - 5}$

Parallel Lines

- Two lines in the same plane that never intersect (symbol for parallel: ll)
- If two lines are parallel, they will have the same slope.

Example 3: Find and use the slopes to determine if line A is parallel to line B.

Line A: Passes through points $(3, 9)$ and $(-5, -1)$

Line B: $f(x) = 4x - 2$

Slope of line A	Slope of line B	Parallel?
$\frac{-1 - -9}{-5 - -3} = \frac{8}{-2}$ $\boxed{m = -4}$	$\boxed{m = 4}$	NO

x y

Example 4: Write the equation for the line that is parallel to $y = \frac{1}{2}x + 7$ passes through the point $(4, 1)$.

$m = \frac{1}{2}$

$y = \frac{1}{2}x + b$

$1 = \frac{1}{2}(4) + b$

$1 = 2 + b$

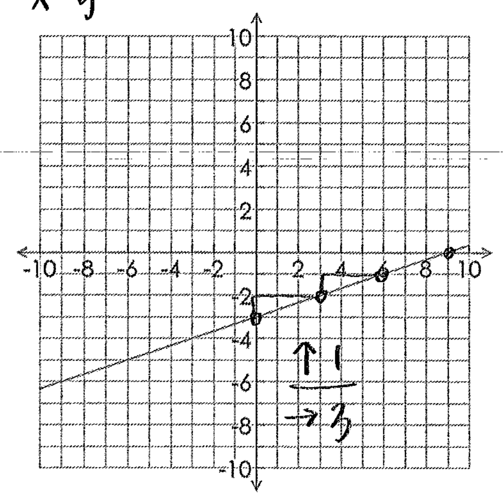
$-2 \quad -2$

$b = -1$

$\boxed{y = \frac{1}{2}x - 1}$

Example 5: Determine the equation of the line parallel to the line graphed below, that passes through the point $(15, 7)$.

x y



$m = \frac{1}{3}$

$y = \frac{1}{3}x + b$

$7 = \frac{1}{3}(15) + b$

$7 = 5 + b$

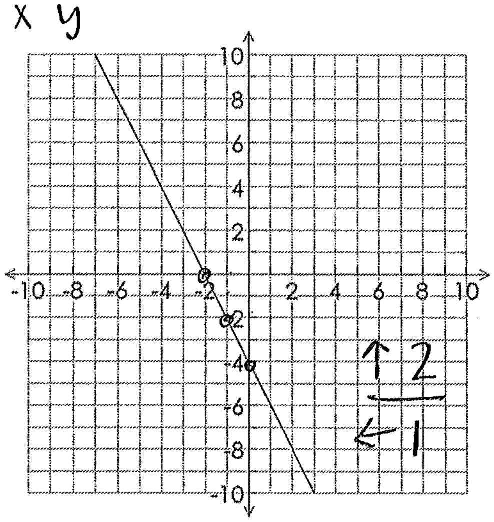
$-5 \quad -5$

$b = 2$

$\boxed{y = \frac{1}{3}x + 2}$

Example 6: Determine the equation of the line parallel to the line graphed below, that passes through the point $(6, -7)$.

x y



$m = -2$

$y = -2x + b$

$-7 = -2(6) + b$

$-7 = -12 + b$

$+12 \quad +12$

$b = 5$

$\boxed{y = -2x + 5}$

Parallel Lines Practice

1. Find and use the slopes to determine if line A is parallel to line B.

   Line A: Has a slope of 2 and passes through point $(5, 6)$ x y

   Line B: Passes through the points $(12,1)$ and $(0, -2)$ x y x y

Line A:
$m = 2$

Line B:
$m = \frac{-2-1}{0-12} = \frac{-3}{-12} = \frac{1}{4}$

$\boxed{\text{No, not parallel}}$

2. Find and use the slopes to determine if line A is parallel to line B.

   Line A: $f(x) = \frac{3}{2}x - 6$

   Line B: Passes through the points $(2,7)$ and $(-4, -2)$ x y x y

$m = \frac{3}{2}$

$m = \frac{-2-7}{-4-2} = \frac{-9}{-6} = \frac{3}{2}$

$\boxed{\text{yes, parallel}}$

3. Determine the equation of a line **parallel** to $y = -8x - 10$ that passes through $(-2, 14)$. x y

$m = -8$

$y = -8x + b$
$14 = -8(-2) + b$
$14 = 16 + b$
$-16 \quad -16$
$b = -2$

$\boxed{y = -8x - 2}$

4. Create the equation of a line **parallel** to $y = \frac{1}{5}x + 2$ that passes through $(20, 13)$. x y

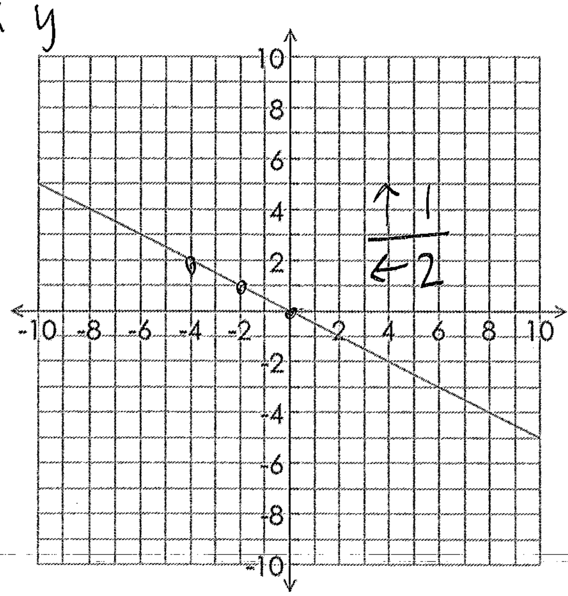
$m = \frac{1}{5}$

$y = \frac{1}{5}x + b$
$13 = \frac{1}{5}(20) + b$
$13 = 4 + b$
$-4 \quad -4$
$b = 9$

$\boxed{y = \frac{1}{5}x + 9}$

5. Determine the equation of the line parallel to the line graphed below, that passes through the point $(10, -8)$.

x y



$m = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$-8 = -\frac{1}{2}(10) + b$

$-8 = -5 + b$

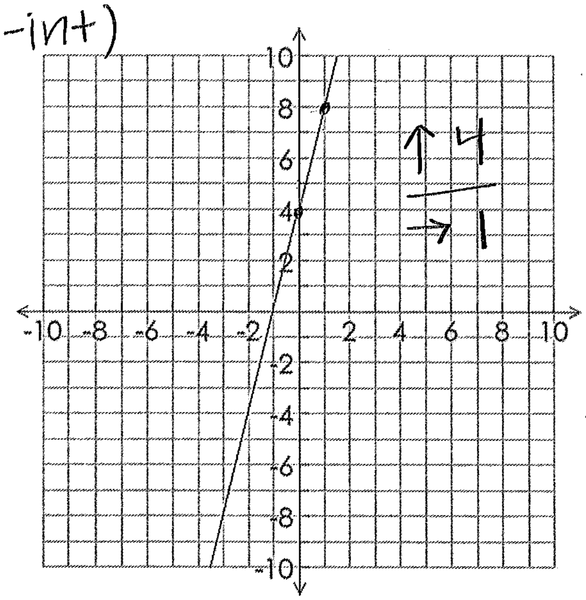
$+5 \quad +5$

$-3 = b$

$\boxed{y = -\frac{1}{2}x - 3}$

6. Determine the equation of the line parallel to the line graphed below, that passes through the point $(0, -1)$.

(y-int)



$m = 4$

$\boxed{y = 4x - 1}$

Perpendicular Lines Notes

Perpendicular Lines

- Two lines in the same plane that intersect to form right angles (symbol for parallel: $\perp$)
- If two lines are perpendicular, they will have <u>opposite</u> <u>reciprocal</u> slopes.
- Hint: Two slopes that are perpendicular will multiply to equal −1. ($\boldsymbol{m \cdot m_{\perp} = -1}$)

Example 1: Determine the slope that would be perpendicular to the following lines.

a. A line with a slope of $\frac{5}{3}$	**b.** A line with a slope of $-\frac{1}{6}$	**c.** A line with a slope of 10
$m_{\perp} = -\frac{3}{5}$	$m_{\perp} = 6$	$m_{\perp} = -\frac{1}{10}$

Example 2: Find and use the slopes to determine if line A is perpendicular to line B.

Line A: Passes through points $\overset{x\ y}{(1,2)}$ and $\overset{x\ y}{(-2,4)}$ **Line B:** $f(x) = \frac{3}{2}x - 1$

Slope of line A	**Slope of line B**	**Perpendicular?**
$\frac{4-2}{-2-1} = \frac{2}{-3}$ $m = -\frac{2}{3}$	$m = \frac{3}{2}$	$\boxed{\text{yes}}$

Example 3: Write the equation of the line perpendicular to $y = -2x + 5$ whose y-intercept is 12.

$m_{\perp} = \frac{1}{2}$ $\boxed{y = \frac{1}{2}x + 12}$

Example 4: Write the equation of the line perpendicular to $y = \frac{1}{5}x - 6$ which passes through the point $\overset{x\ y}{(1,-3)}$.

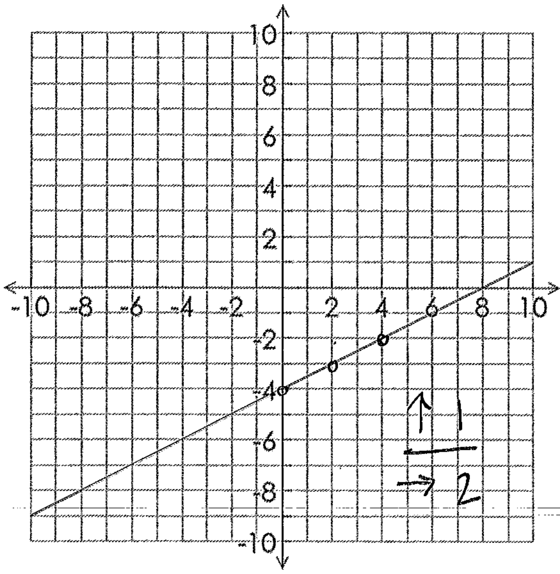
$m_{\perp} = -5$

$y = -5x + b$
$-3 = -5(1) + b$
$-3 = -5 + b$
$+5 \quad +5$
$b = 2$

$\boxed{y = -5x + 2}$

Example 5: Determine the equation of the line perpendicular to the line graphed below, that passes through the point $(10, -13)$.

x y



$m = \frac{1}{2}$

$\perp\ m = -2$

$y = -2x + b$

$-13 = -2(10) + b$

$-13 = -20 + b$

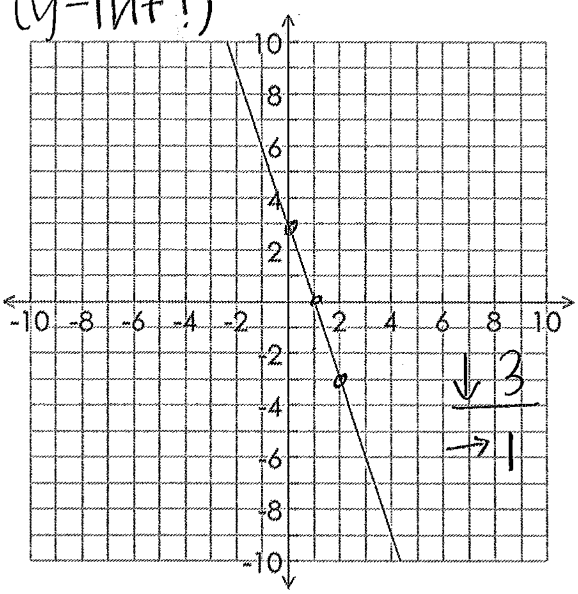
$+20 \quad +20$

$b = 7$

$\boxed{y = -2x + 7}$

Example 6: Determine the equation of the line perpendicular to the line graphed below, that passes through the point $(0, -6)$.

(y-int!)



$m = -3$

$\perp\ m = \frac{1}{3}$

$\boxed{y = \frac{1}{3}x - 6}$

Perpendicular Lines Practice

1. Find and use the slopes to determine if line A is perpendicular to line B.

   x y x y

   Line A: A line that passes through the points $(6,-8)$ and $(0,2)$

   Line B: $g(x) = \frac{3}{5}x - 9$

Line A:

$m = \frac{2--8}{0-6} = \frac{10}{-6}$

$m = \frac{-5}{3}$

Line B:

$m = \frac{3}{5}$

$\boxed{\text{yes, perpendicular}}$

2. Find and use the slopes to determine if line A is perpendicular to line B.

   Line A: Has a slope of 7 and passes through point $(3,20)$

   x y x y

   Line B: Passes through the points $(0,-1)$ and $(-2,13)$

Line A:

$m = 7$

Line B:

$m = \frac{13--1}{-2-0} = \frac{14}{-2}$

$m = -7$

$\boxed{\text{NO, not perpendicular}}$

x y

3. Develop the equation of a line **perpendicular** to $y = -\frac{2}{5}x - 1$ to that passes through $(2,-8)$.

$\perp\ m = \frac{5}{2}$

$y = \frac{5}{2}x + b$

$-8 = \frac{5}{2}(2) + b$

$-8 = 5 + b$

$-5 \quad -5$

$-13 = b$

$\boxed{y = \frac{5}{2}x - 13}$

x y

4. Decide the equation of a line **perpendicular** $y = -\frac{1}{2}x - 6$ to that passes through $(4,5)$.

$\perp\ m = 2$

$y = 2x + b$

$5 = 2(4) + b$

$5 = 8 + b$

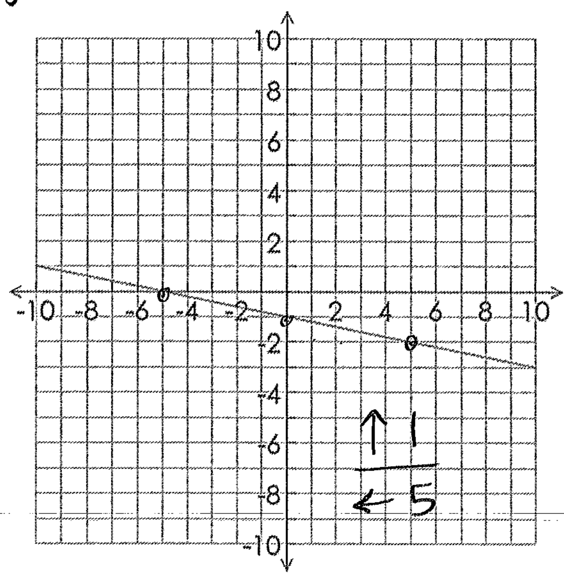
$-8 \quad -8$

$b = -3$

$\boxed{y = 2x - 3}$

5. Determine the equation of the line perpendicular to the line graphed below, that passes through the point (3,21).

x y



$m = -\frac{1}{5}$

$\perp m = 5$

$y = 5x + b$

$21 = 5(3) + b$

$21 = 15 + b$

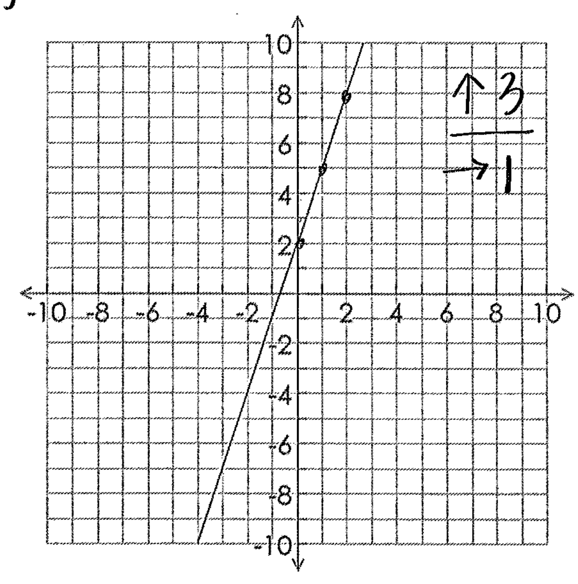
$-15 \quad -15$

$6 = b$

$\boxed{y = 5x + 6}$

6. Determine the equation of the line perpendicular to the line graphed below, that passes through the point (3, 2).

x y



$m = 3$

$\perp m = -\frac{1}{3}$

$y = -\frac{1}{3}x + b$

$2 = -\frac{1}{3}(3) + b$

$2 = -1 + b$

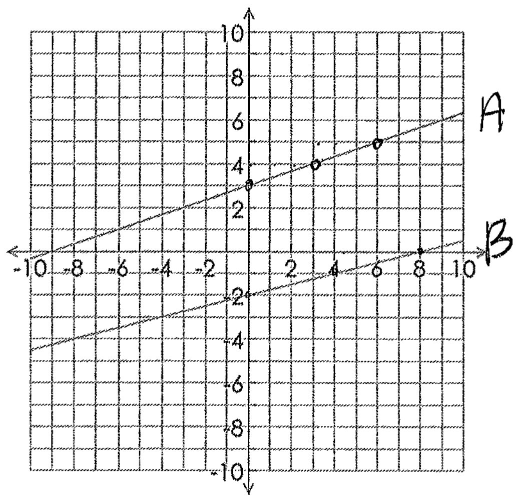
$+1 \quad +1$

$b = 3$

$\boxed{y = -\frac{1}{3}x + 3}$

Parallel and Perpendicular Lines Practice

1. Determine if the lines graphed below are parallel. Explain your reasoning.

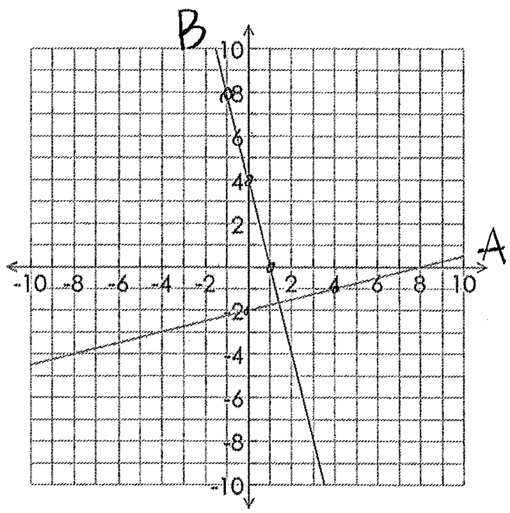


line A: $m = \frac{1}{3}$

line B: $m = \frac{1}{4}$

No, not parallel - they don't have the same slope.

2. Determine if the lines graphed below are perpendicular. Explain your reasoning.

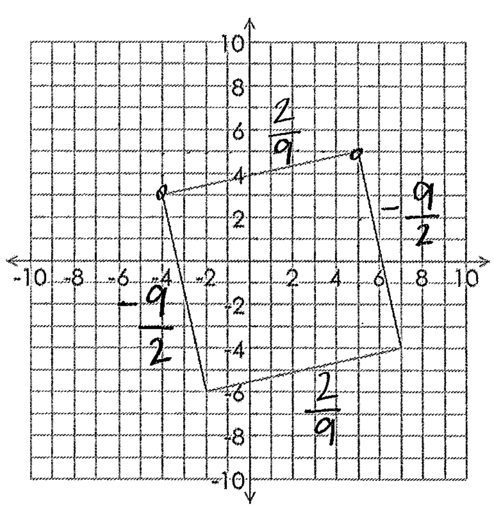


line A: $m = \frac{1}{4}$

line B: $m = -4$

yes, they are perpendicular - they have opposite, reciprocal slopes.

3. All the sides of the shape graphed below have the same length. Shelby is convinced that the shape is a square. Amanda disagrees and thinks that it is a rhombus. Explain who is correct and why.



Shelby is correct. A square has 4 right angles and the slopes show that the lines are perpendicular.

4. A racing company has a drag strip that begins at the point $(-4, 19)$ and ends at the point $(8, -17)$. The company wants to build another drag strip so that it is parallel to the current drag strip and passes through the point $(1,3)$. What is the equation for the new drag strip?

original

$m = \frac{-17-19}{8--4} = \frac{-36}{12}$

$m = 3$

$y = 3x + b$

$3 = 3(1) + b$

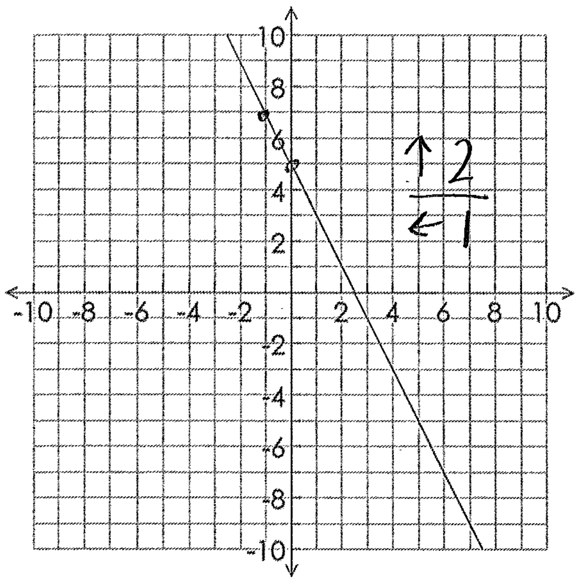
$3 = 4 + b$

$-4 \quad -4$

$b = -1$

$\boxed{y = 3x - 1}$

5. The location of a road for a new town is shown on the coordinate plane below. Two additional roads are planned: one that will be parallel and one that will be perpendicular to the road below. Each will pass through the point $(-6, 8)$ on the coordinate plane. Determine the equations of the roads.



parallel:

$y = -2x + b$

$8 = -2(-6) + b$

$8 = 12 + b$

$-12 \quad -12$

$b = -4$

$\boxed{y = -2x - 4}$

perpendicular:

$y = \frac{1}{2}x + b$

$8 = \frac{1}{2}(-6) + b$

$8 = -3 + b$

$+3 \quad +3$

$11 = b$

$\boxed{y = \frac{1}{2}x + 11}$

6. A driveway is being built in a new neighborhood and has a slope of 2 and passes through the point $(5,7)$. If you want to build another driveway that is exactly one mile away (1 unit = 1 mile), what is a possible equation for the new driveway?

$y = 2x + b$

$7 = 2(5) + b$

$7 = 10 + b$

$-10 \quad -10$

$-3 = b$

$y = 2x - 3$

$\boxed{\begin{gathered} y = 2x - 4 \\ \text{or} \\ y = 2x - 2 \end{gathered}}$

Distance Formula Notes

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Where (x_1, y_1) and (x_2, y_2) are the endpoints of the segment and d is the distance of the segment.

Example 1: Determine the distance between the points $(-5, 3)$ and $(-1, 1)$. Give your answer in both exact form and rounded to the nearest tenth.

d = $2\sqrt{5}$ units

d ≈ 4.5 units

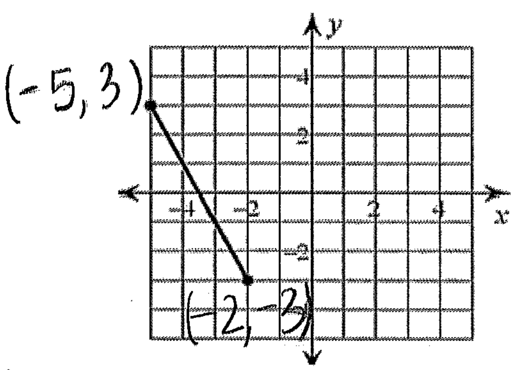
$d = \sqrt{(-1 - -5)^2 + (1 - 3)^2}$

$d = \sqrt{20} = 2\sqrt{5} \approx 4.5$

$\boxed{5}$ 4

2 2

Example 2: Calculate the distance of the line segment graphed below. Give your answer in both exact form and rounded to the nearest tenth.



$d = \sqrt{(-2 - -5)^2 + (-3 - 3)^2}$

$d = \sqrt{45} = 3\sqrt{5}$

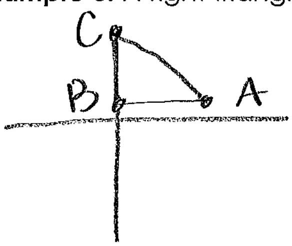
9 $\boxed{5}$

3 3

d = $3\sqrt{5}$ u

d ≈ 6.7 u

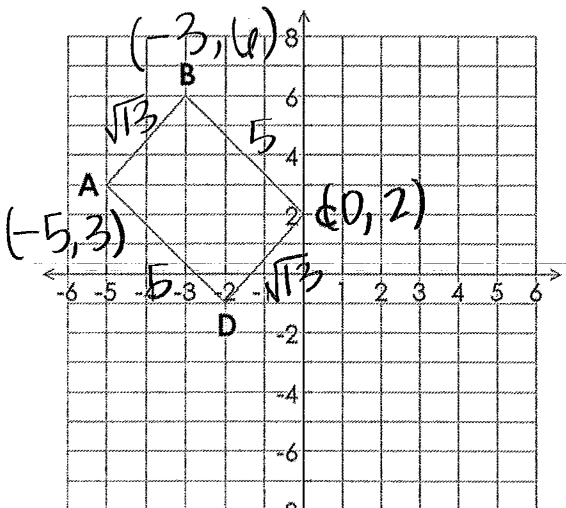
Example 5: A right triangle has vertices at $A(3,1)$, $B(0,1)$, and $C(0,9)$. What is the distance of the hypotenuse?



$$d = \sqrt{(0-3)^2 + (9-1)^2}$$

$$\boxed{d = \sqrt{73}\ u}$$

Example 6: Find the perimeter of the rectangle below. Give your answer in both exact form and rounded to the nearest tenth.



$$\overline{AB}: d = \sqrt{(-5 - -3)^2 + (3-6)^2}$$

$$d = \sqrt{13}$$

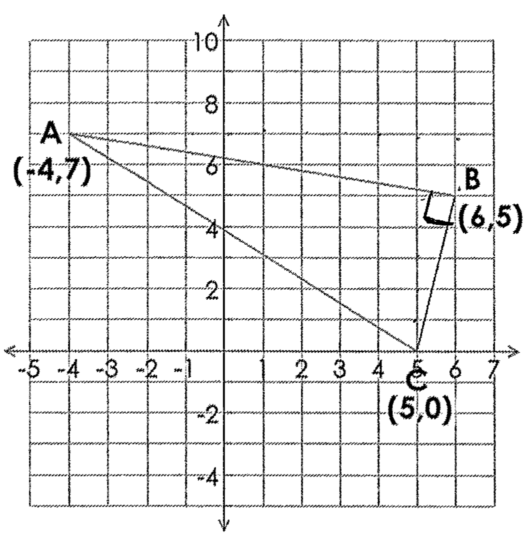
$$\overline{BC}: d = \sqrt{(-3-0)^2 + (2-6)^2}$$

$$d = \sqrt{25} = 5$$

$$\text{perimeter} = \sqrt{13} + 5 + \sqrt{13} + 5$$

Perimeter = $10 + 2\sqrt{13}\ u$ Perimeter ≈ $17.2\ u$

Example 7: Using the formula $A = \frac{1}{2}bh$ for the area of a triangle, determine the area of the shape below.



$$\left.\begin{array}{l} m(AB) = -\frac{1}{5} \\ m(BC) = 5 \end{array}\right\}\ \text{perpendicular}$$

$$\overline{AB}: d = \sqrt{(-4-6)^2 + (7-5)^2}$$

$$d = \sqrt{104}$$

$$\overline{BC}: d = \sqrt{(6-5)^2 + (5-0)^2}$$

$$d = \sqrt{26}$$

Area = $26\ u^2$

$$\text{Area} = \frac{1}{2} \cdot \sqrt{104} \cdot \sqrt{26}$$

$$= \frac{1}{2} \cdot 52$$

Distance Formula Practice

1. Determine the distance between the points ~~(-5, 8)~~ and ~~(-1, 1)~~. Give your answer in both exact form and rounded to the nearest tenth.

x y x y
(6, −4) (2, 4)

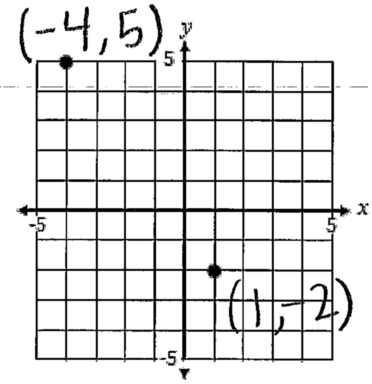
d = $4\sqrt{5}$ u

d ≈ 8.9 u

$d = \sqrt{(2-6)^2 + (4 - -4)^2} = \sqrt{80}$

8 10
4 2 2 $\boxed{5}$
2 2

2. Determine the distance between the two given points. Give your answer in both exact form and rounded to the nearest tenth.



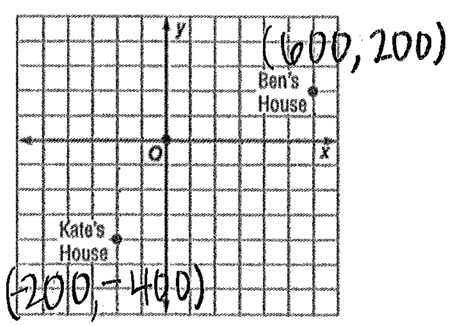
d = $\sqrt{74}$ u

d ≈ 8.6 u

$d = \sqrt{(-4-1)^2 + (5 - -2)^2} = \sqrt{74}$

2 37

3. Ben and Kate are making a map of their neighborhood on a piece of graph paper. They decide to make one unit on the graph paper correspond to 100 yards. First, they put their homes on the map as shown below. How far apart are their homes?

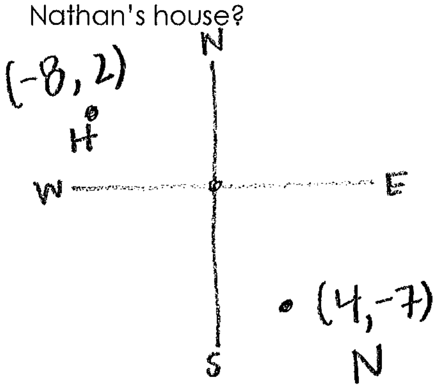


$d = \sqrt{(600 - -200)^2 + (200 - -400)^2}$

$d = \sqrt{1000000}$

$\boxed{d = 1{,}000 \text{ yards}}$

4. Milton High School is located at the point (0,0). Harrison lives 8 miles west and 2 miles north of the school. Nathan lives 4 miles east and 7 miles south of the school. What is the distance between Harrison and Nathan's house?

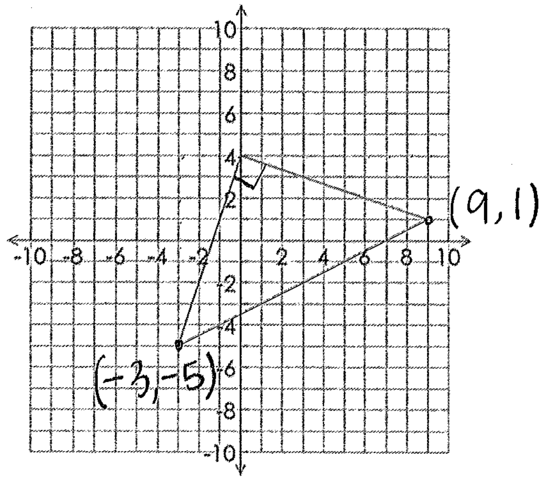


$d = \sqrt{(-8-4)^2 + (2 - -7)^2}$

$d = \sqrt{225}$

$d = \boxed{15 \text{ miles}}$

5. Using the graph below, determine the length of the hypotenuse of the right triangle. Give your answer in both exact form and rounded to the nearest tenth.



$d = \sqrt{(9 - {}^-3)^2 + (1 - {}^-5)^2}$

$d = \sqrt{180}$

180: 18, 10

18: 9, 2 ; 10: 2, $\boxed{5}$

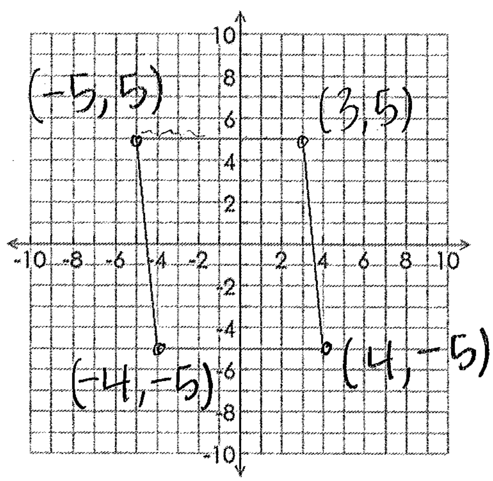
9: 3 3

$= 6\sqrt{5}$

hypotenuse = $6\sqrt{5}$ u

hypotenuse ≈ 13.4 u

6. Calculate the perimeter of the shape below. Give your answer in both exact form and rounded to the nearest tenth.



$d = 8$

$d = 8$

$d = \sqrt{(3-4)^2 + (5 - {}^-5)^2} = \sqrt{101}$

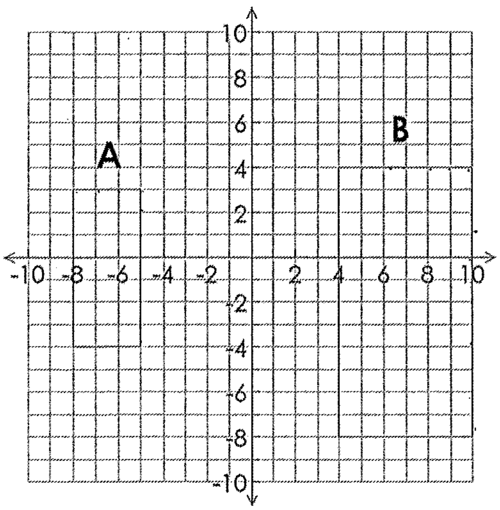
$d = \sqrt{(-5 - {}^-4)^2 + (5 - {}^-5)^2} = \sqrt{101}$

perimeter $= 8 + 8 + \sqrt{101} + \sqrt{101}$

Perimeter = $16 + 2\sqrt{101}$ u

Perimeter ≈ 36.1 u

7. Find the perimeter of the two rectangles graphed below. How do their perimeters compare to each other?



A: $3 + 7 + 3 + 7$

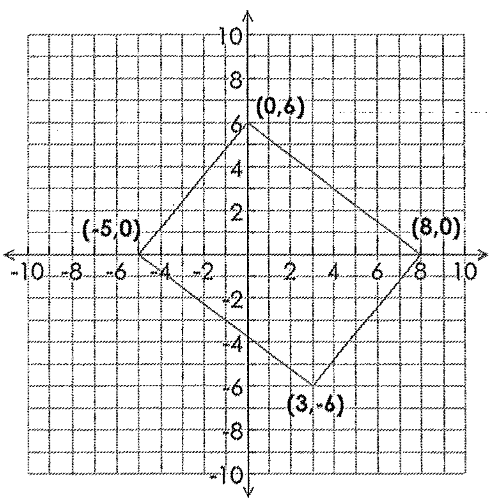
$p = 20$

B: $6 + 14 + 6 + 14$

$p = 40$

Rectangle B is twice the size of rectangle A.

8. Determine the area of the rectangle.



area $= b \cdot h$

$d = \sqrt{(8-0)^2 + (0-6)^2} = 10$

$d = \sqrt{(8-3)^2 + (0-6)^2} = \sqrt{61}$

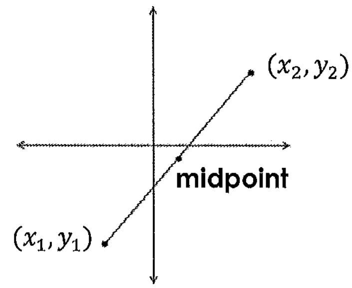
area $= 10 \cdot \sqrt{61}$

$= \boxed{10\sqrt{61} \text{ units}^2}$ or $\boxed{78.1 \text{ units}^2}$

Midpoint Formula Notes

The midpoint is the point that divides a segment in half.

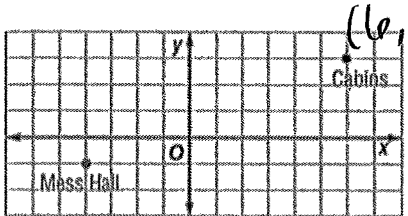
$$\text{Midpoint Formula: } \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$



Example 1: Find the midpoint of $\overline{GH}$ given the endpoints $G(7, -5)$ and $H(10, -1)$.

$$\left(\frac{7+10}{2}, \frac{-5+-1}{2}\right) = \boxed{(8.5, -3)}$$

Example 2: Troop 175 is designing their new campground by first mapping everything on a coordinate grid. They have found a location for the mess hall and for their cabins. They want the bathrooms to be halfway between these two. What will be the coordinates of the location of the bathrooms?



$$\left(\frac{6+-4}{2}, \frac{3+-1}{2}\right)$$

$$\boxed{(1, 1)}$$

Example 3: Using the same image above, Troop 175 wants to also put in a storage building for all their lake equipment. They want the cabins to be halfway between the bathrooms and the storage building. What is the coordinate location of the storage building?

bathrooms (1, 1)   cabins (6, 3) $\boxed{\text{storage } (11, 5)}$

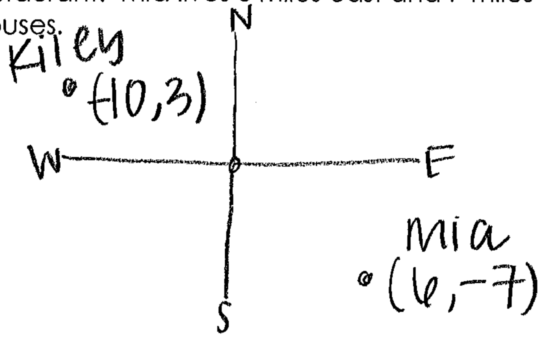
+5   +2

Example 4: The grocery store represents the midpoint between Wills Park and Top Golf. If the grocery store is at the point $(4, -3)$ and Wills Park is located at $(-2, 7)$, what are the coordinate of Top Golf?

Wills park (-2, 7)   Grocery Store (4, -3) $\boxed{\text{Top Golf } (10, -13)}$

+6   -10

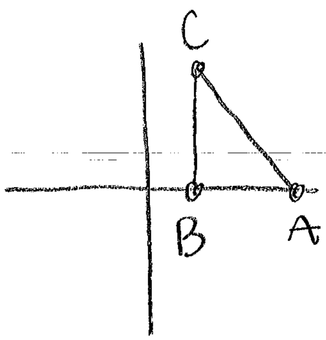
Example 5: Old Blind Dog is located at the point (0, 0). Kiley lives 10 miles west and 3 miles north of the restaurant. Mia lives 6 miles east and 7 miles south of the restaurant. Determine the midpoint between their houses.



$$\left(\frac{-10+6}{2}, \frac{3+-7}{2}\right)$$

$$\boxed{(-2,-2)}$$

Example 6: A triangle has vertices of $A(9,0)$, $B(3,0)$, and $C(3,14)$. Determine the midpoint of the longest side of the triangle.



$$\left(\frac{9+3}{2}, \frac{0+14}{2}\right)$$

$$\boxed{(6,7)}$$

Example 7: Two cruise ships started at the same point and went in opposite directions at the same speed. After one-hour Royal Caribbean was at point $(17,33)$ and the Harmony of the Seas was at the point $(-19,-39)$. At what point did they start?

Start

$(17,33) \longleftarrow \circ \longrightarrow (-19,-39)$

$$\left(\frac{17+-19}{2}, \frac{33+-39}{2}\right)$$

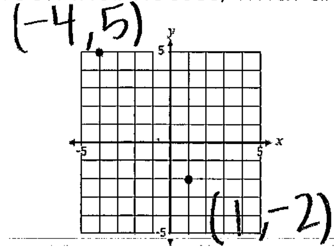
$$\boxed{(-1,-3)}$$

Midpoint Formula Practice

1. Find the midpoint between the points (−5, 7) and (3, −1).

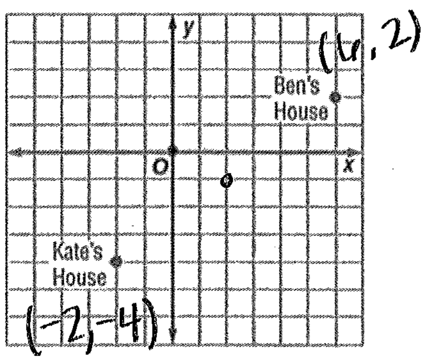
$$\left(\frac{-5+3}{2}, \frac{7+-1}{2}\right) = \boxed{(-1,3)}$$

2. Casey and Ross's homes are shown on the coordinate grid below. If they want to meet exactly halfway between their houses, what are the coordinates of the point that represents this location?



$$\left(\frac{-4+1}{2}, \frac{5+-2}{2}\right) = \boxed{(-1.5, 1.5)}$$

Ben and Kate are making a map of their neighborhood on a piece of graph paper. ~~They decide to make one unit on the graph paper correspond to 100 yards~~. First, they put their homes on the map as shown below.



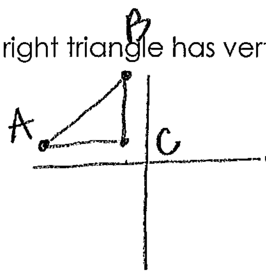
3. Their friend Jason lives exactly halfway between Ben and Kate. Determine the coordinate location of Jason's house?

$$\left(\frac{6+-2}{2}, \frac{2+-4}{2}\right) = \boxed{(2,-1)}$$

4. Kate lives halfway between Jason and Marie. What is the coordinate location of Marie's house?

Jason (2,−1) Kate (−2,−4) **Marie (−6,−7)**

5. A right triangle has vertices of $A(-8,1)$, $B(-2,13)$ and $C(-2,1)$. Determine the midpoint of the hypotenuse.



$\overline{AB}$ is hypotenuse

$$\left(\frac{-8+-2}{2}, \frac{1+13}{2}\right) = \boxed{(-5,7)}$$

6. On a map's coordinate grid, Alpharetta is located at (18, −24) and Milton is located at (32, −6). If Alpharetta is exactly halfway between Milton and Roswell, determine the coordinate location of Roswell.

Milton (32, −6) Alph. (18, −24) **Roswell (4, −42)**